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Deformed Symmetries from Quantum Relational Observables

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Abstract. Deformed Special Relativity (DSR) is a candidate phenomenological theory to describe the Quantum Gravitational (QG) semi-classical regime. A possible interpretation of DSR can be derived from the notion of deformed reference frame. Observables in (quantum) General Relativity can be constructed from (quantum) reference frame – a physical observable is then a relation between a system of interest and the reference frame. We present a toy model and study an example of such quantum relational observables. We show how the intrinsic quantum nature of the reference frame naturally leads to a deformation of the symmetries, comforting DSR to be a good candidate to describe the QG semi-classical regime.

Introduction

There are currently many hopes that QG phenomena could be measured in different contexts — e.g. either from astrophysical observation if the effects are additive or in particle accelerators if space-time has extra dimensions. More concretely, GLAST will measure possible differences in the time of arrival of γ-ray bursts which could be related to the quantum structure of spacetime. Data should be out as soon as 2008. It is therefore important to construct consistent theories to predict these data. One would like to consider a partition function containing both matter and gravitational degrees of freedom, then integrate out the gravitational degrees of freedom to obtain the effective lagrangian describing matter, affected by the QG fluctuations around a chosen spacetime (usually Minkowski spacetime):

$$\int [d\phi_M][dg] e^{i\int \mathcal{L}_M(\phi_M,g)+\mathcal{L}_{GR}(g)} \rightarrow \int [d\phi_M] e^{i\int \tilde{\mathcal{L}}_M(\phi)}, \tag{1}$$

where $\mathcal{L}_M(\phi_M, g)$, $\mathcal{L}_{GR}(g)$ are lagrangians for respectively matter and gravitational degrees of freedom. More generally, integrating out gravitational degrees of freedom might lead to a

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stochastic dynamics of the matter fields, i.e. a dynamics taking pure states to mixed states and so with a finite entropy growth.

Performing this calculation explicitly of course requires a good handling of quantized gravity, as well as a clear definition of matter in this extreme regime. Depending on the candidate theory of Quantum Gravity one has faith in, (e.g. extra dimensions/strings/branes, or loop quantum gravity/spinfoam models, etc.), one can try to perform the derivation more or less rigorously. For example, in the case of extra dimensions, it has been argued (see [1]) that gravitons can evade in extra dimensions and thus generate a Lorentz symmetry breaking when their contribution is integrated out and projected on the 3-brane.

In the context of the Canonical Quantization scheme, the derivation has been made rigourously only in 3 spacetime dimensions [2], using the Ponzano-Regge model to “regularize” the gravitational contribution. The resulting effective theory is an example of Deformed Special Relativity (DSR) [3]. The 4 dimensional theory is not purely topological and so much harder to handle exactly; the idea instead is to construct the effective theory describing these QG degrees of freedom by hand. In particular, one would like to modify the notion of (Minkowski) spacetime to take into account some QG features. Since Minkowski spacetime is defined in terms of its symmetries – namely the Poincaré symmetries – it is natural to attempt to incorporate QG phenomenology by modifying these symmetries. There are at least two ways to proceed: one can either break or deform the symmetries. Strictly speaking, a deformation also implies breaking the symmetry since the original symmetry is no longer realized. However, there appears to be a key distinction between the two situations: the broken symmetry case allows for new type of interactions forbidden in the original theory [4]. In case of a Lorentz violation for instance, photons can decay. It is not absolutely clear whether this can occur in the case of a deformed symmetry since a full (quantum) field theory is still lacking. However adopting the point of view that deformed symmetries can be constructed using a deformation map [5], so that multiparticle states can always be traced back to an undeformed case, it is clear that these Lorentz violation interactions cannot occur.

A prime motivation for modifying the Poincaré symmetries is to introduce the Plank scale in the theory either as a maximum bound M_P of the 3 dimensional momentum or as a minimal length L_P for the space length. The idea of a minimal/maximal scale clearly indicates that the Lorentz boosts are to be altered in some way.

There is an extensive literature on Lorentz symmetry breaking and strong experimental constraints on it e.g. from astrophysical data [6]. At this stage it appears unlikely that such Lorentz symmetry breaking can correctly describe low energy QG. Here, with the help of a simple model, we wish to argue that one should instead expect a deformation of the symmetries. As a first step, we will review the characteristics of such a deformed symmetry. In particular, we will emphasize how a deformation of the symmetries implies a modified notion of reference frame, and *vice versa*. We then present our model in which we can readily identify the relational observables and, drawing upon previous work [7, 8], outline how it leads to a symmetry deformation. The full details of this model will be presented in a forthcoming paper [9].

1. Deformed symmetries

Deformed symmetries are usually constructed in phase space, that is the cotangent bundle. To understand what geometry is associated with this deformed structure, one performs a Legendre transform to move to the tangent bundle and obtain a Finsler geometry [10].

In DSR, the Plank mass in introduced in momentum space as a universal maximum value of either the rest mass, energy, or 3 dimensional momentum. Often the deformation of the symmetry is constructed solely in momentum space, without concerns for the consequences on spacetime. One assumes the existence of an auxiliary momentum variable π carrying a linear

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representation of the Lorentz group. One then introduces a *invertible* "deformation map"

$$U_{M_P}(\pi) = p \tag{2}$$

relating the auxiliary momentum to the physical momentum. The action of the Lorentz group on p is then non-linear (though a suitable choice of U can leave the rotations unaffected [5]):

$$p \to U_{M_P}\left(\Lambda \cdot U^{-1}_{M_P}(p)\right), \tag{3}$$

where $\Lambda \in SO(3,1)$. The Casimir associated to these deformed symmetries can then be written as

$$E^2 = m^2 + p^2 + F(p, \mu, M_{\rm P}), \tag{4}$$

where F is a function of dimension mass^2, μ is a possible set of extra mass parameters (like Higgs mass...), and $p = |\vec{p}|$. Natural questions in this setting include what is the physical meaning of the auxiliary momentum and also what is spacetime.

To get more information about spacetime, one can try to represent the Lorentz algebra in different ways (see for example [11]). The non-linear realization in (3) means that the boosts do not act in the usual way on momentum:

$$[N_i, p_j] = Ap_0\delta_{ij} + Bp_ip_j + C\epsilon_{ijk}p_k + ..., \qquad [N_i, p_0] = Dp_i + ..., \tag{5}$$

where A, B, C, D are functions of p_0, p_i^2, M_P, such that the limit $M_P \to \infty$ leads to the usual action of the boost on momentum. If representing the boosts as $N_i = x_0p_i + x_ip_0$, one needs a non-trivial symplectic form in order for (5) to hold true. Another possibility is to keep the usual trivial symplectic form and write a deformed boost as $\tilde{N}_i = N_i + f(x, p)$, such that the Lorentz algebra together with (5) is still fulfilled (this is the point of view of [5]). Finally, there is also the possibility of having both of these pictures combined: a deformed boost together with a non-trivial symplectic form. Note that a modified symplectic form leads to modified uncertainty relations [12].

To construct multi-particle states, one needs to define the addition of momenta. There is a lot of freedom in defining this addition. It could be non commutative [11] (sometimes even non associative), e.g.

$$p_1^0 \oplus p_2^0 = p_1^0 + p_2^0,\ p_1^i \oplus p_2^i = p_1^i + e^{-p_1^0}p_2^i. \tag{6}$$

Another possibility is to use the auxiliary momentum to define the addition, in which case the addition is commutative [5]

$$p_1 \oplus p_2 = U_{\alpha M_P}\left(U^{-1}_{M_P}(p_1) + U^{-1}_{M_P}(p_2)\right). \tag{7}$$

The α factor comes into play to avoid the soccer ball problem [5, 13], which in general plagues the first type of addition.

To sum up, we see that in the absence of further physical guidance, there is a lot of freedom in the physics associated to the deformed symmetries. One can gain some insight by adopting an operational point of view and study the problem from the perspective of the reference frames that are intimately related to the effective symmetries of the system.

2. Modified reference frame

In order to explain the physics of (4), Liberati et al. proposed a modified notion of reference frame [14]. In this case, the auxiliary momentum π_μ acquires a physical significance: it is the intrinsic momentum of the particle. To measure it, we need to introduce a reference frame ${e^\mu}_\alpha$, which is identified with the tetrad [15]. The μ's are space-time indices and are transforming as tensor indices. The outcome of the measurement are scalars p_α, obtained upon projection of π on the reference frame e

$$p_\alpha = \pi_\mu {e^\mu}_\alpha. \tag{8}$$

In the Minkowski case, the tetrad is trivial so that ${e^\mu}_\alpha \sim {\delta^\mu}_\alpha$, this just means that π and p coincide. In this simple case, upon change of reference frame, p transforms linearly under Lorentz transformation, since different reference frames are simply related by some Lorentz transformation

$$p'_\alpha = \pi_\mu {\bar{e}^\mu}_\alpha = {\Lambda^\beta}_\alpha {e^\mu}_\beta \pi_\mu = {\Lambda^\beta}_\alpha p_\beta. \tag{9}$$

We naturally have a linear realization of the Lorentz symmetries.

Liberati et al. provided different arguments how effective treatments of the (quantum) gravitational fluctuations can generate a non-trivial mixing between the reference frame and the particle, leading to a non-trivial notion of reference frame [14]. For example, naively, the tetrad could also encompass the gravitational field generated by the (quantum) particle (which is usually neglected) and so be dependent on the particle momentum. The outcome of the measurement p_α is now a non-linear *invertible* function U_{M_P} of the intrinsic momentum π

$$p = U_{M_P}(\pi \cdot e) \sim \pi_\mu {e^\mu}_\alpha(\pi), \tag{10}$$

and this dependence becomes trivial in the limit of infinite M_P. Upon change of reference frame under Lorentz transformation, p_α will clearly transform non-linearly as in (3), thus inducing a non-linear Lorentz transform

$$\tilde{\Lambda} \cdot p = U_{M_P}\left(\Lambda \cdot U^{-1}_{M_P}(p)\right). \tag{11}$$

The case of multiparticle states has not been studied in great details in this scheme. However it seems natural to take the intrinsic momentum of a 2-particles system to be $\pi_{tot} = \pi_1 + \pi_2$. In the usual Minkowski case, the measured total momentum is given by

$$p_{tot} = (\pi_1 + \pi_2)^\mu \, e_\mu = p_1 + p_2. \tag{12}$$

Taking into account the gravitational "kick back" however, the reference frame becomes system dependent, and we obtain instead

$$p_{tot} = (\pi_1 + \pi_2)^\mu \, e_\mu(\pi_1 + \pi_2) = U_{\alpha M_P}(\pi_1 + \pi_2) = U_{\alpha M_P}\left(U^{-1}_{M_P}(p_1) + U^{-1}_{M_P}(p_2)\right). \tag{13}$$

We recognize the addition defined in (7). The use of reference frames is natural in the context of a diffeomorphism invariant theory. They are the basic tool to construct observables in the General Relativity context. More generally, they can also be used to construct observables in a (quantum) constrained theory.

3. Observables in a constrained toy model

GR is an example of a constrained theory: working in the hamiltonian formalism, there is a set of first class constraints [16] that encodes the diffeomorphisms symmetry. Observable quantities are functions on phase space that commute with the constraints. It is in general very hard to construct a complete set of observables. Nonetheless, a large class of observables can typically be constructed from relations between systems and reference frames [17, 18] (see [19] for a recent perspective).

In the quantum regime, we also expect observables to be given in terms of relations to quantum reference frames. Here we will use a very simple model that illustrates this general construction, and may serve as a benchmark of the general ideas presented in [14] and outlined in Sec. 2. The full details of the model will be presented elsewhere [9] we only give the general outlines here. This model has been studied in the context of relational quantum mechanics in [21] and also as a model of high precision measurement in the context of Quantum Information Theory [20, 8].

Consider a universe made of N quantum spins-$\frac{1}{2}$ particles (qubits). The kinematics is specified in terms of the Pauli matrices $\vec{\sigma}^i$ where we use the shorthand $\vec{\sigma}^i \equiv \mathbb{1} \otimes ... \otimes \vec{\sigma} \otimes \mathbb{1} \otimes ...$ to denote the Pauli matrices acting on the ith particle. This universe is rotationally invariant and so must satisfy the constraint

$$\left(\sum_i^N \sigma_x^i\right)^2 + \left(\sum_i^N \sigma_y^i\right)^2 + \left(\sum_i^N \sigma_z^i\right)^2 = 0, \tag{14}$$

(see [21] for a discussion on possible relaxation of this constraint). Observables are quantities that are invariant under global rotations. A natural candidate is the length of the vector, $\vec{\sigma}^2$. A more interesting example is the relative angle between two vectors, which is clearly invariant under global rotations. Inspired by how spins are measured in real-life experiments, we can construct a reference frame out of a large number of particles and measure the relative orientation of a qubit with respect to this reference frame. Since every object in this toy Universe is quantized, we will thus obtain a Quantum Reference Frame (QRF) which will naturally be subject to quantum fluctuations. We choose two non-intersecting sets of qubits, $\mathcal{R}_1,\ \mathcal{R}_2$, containing respectively $R_1,\ R_2$ qubits and define the operators

$$\vec{J}_{\mathcal{R}_i} = \sum_{l \in \mathcal{R}_i} \vec{\sigma}_l,\ \forall i = 1, 2. \tag{15}$$

We define the normalized operators $\vec{J}_i = \vec{J}_{\mathcal{R}_i}(\vec{J}^2_{\mathcal{R}_i})^{-\frac{1}{2}}$, for $j = 1, 2$ using the generalized inverse. This is well defined since $\vec{J}^2_{\mathcal{R}_i}$ is a Casimir and so commutes with $\vec{J}_{\mathcal{R}_i}$. The full reference frame, noted $\mathcal{J}$, is now given by $J_j{}^a = \{J_1{}^a, J_2{}^a, J_3{}^a = \epsilon^a_{bc} J_1^b J_2^c\}$, and plays a similar role as the tetrad field $e_\mu{}^a$.

For every particle $k \notin \mathcal{R}_1,\ \mathcal{R}_2$, we construct a set of "relational Pauli operators"

$$s_j^k = \vec{\sigma}^k \cdot \vec{J}_j \tag{16}$$

which are clearly invariant under global rotations. They can be interpreted as the coordinates of $\vec{\sigma}^k$ in the reference frame $\mathcal{J}$. Note also that $\vec{\sigma}^k$ can be identified with π, the intrinsic quantity, so that $\vec{s}$ is analog to p.

One can straightforwardly verify that the physical symplectic form is altered by this definition[3]

$$[s_j^k, s_l^k] \neq \epsilon_{jl}{}^m s_m^k. \tag{17}$$

[3] Note that this is not due to the "non-orthogonality" of the reference frame axis. We can use a Gram-Schmidt procedure to make sure that the J_j are mutually orthogonal and still get a similar effect.

However the symplectic structure is completely fixed by our choice of relational observables, so that there is no ambiguity as mentioned in Sec. 1. The spectrum of the operators is also altered. Assume for simplicity that the particles of $\mathcal{R}_j$ satisfy $(\vec{J}_{\mathcal{R}_j})^2 = n(n+1)$ for some n. Subjected to that constraint, s_j^k has two distinct eigenvalues

$$\lambda_j^- = -\frac{1}{2}\frac{n+1}{\sqrt{n(n+1)}} \quad \text{and} \quad \lambda_j^+ = \frac{1}{2}\frac{n}{\sqrt{n(n+1)}} \tag{18}$$

which differ from the $\pm\frac{1}{2}$ eigenvalues of the kinematic observables by a correction of order n^{-1}. As in DSR, the multiparticle operators are also significantly different. At the kinematic level, we have $[\sigma_i^k, \sigma_j^l] = 0$ for $k \neq l$ which also implies that the spin operator of a pair of particle $\vec{\Sigma}^{(k,l)} = \vec{\sigma}^k \otimes \mathbb{1} + \mathbb{1} \otimes \vec{\sigma}^m$ commutes with that of the individual components: $[\Sigma_j^{(k,l)}, \sigma_j^l] = 0$. Both of these properties are modified at the level of relational observables. The total physical coordinate

$$S_j^{(k,l)} = \vec{\Sigma}^{(k,l)} \cdot \vec{J}_j = \left(\vec{\sigma}^k \otimes \mathbb{1} + \mathbb{1} \otimes \vec{\sigma}^m\right) \cdot \vec{J}_j \tag{19}$$

does not commute with s_j^k nor s_j^l, so measuring the total spin of two particles is quite different that measuring their individual spins and adding the components. This is a straightforward consequence of the fact that $[s_j^k, s_j^l] \neq 0$ and so the spin measurement on distinct particles disturb one another.

Despite these important distinctions, the relational and kinematic Pauli operators can operationally behave quite similarly in the appropriate "semi-classical" regime. For instance, we see from (18) that the spectrum of say σ_1 and s_1 match in the limit $j \to \infty$. To further analyze the operational relation between σ_1 and s_1, it is best not to focus on the observables themselves but rather on the *measurement they induced on particle k*. The spectral decomposition of s_1 subjected to the above constraint has two distinct projectors $\Pi_1^\pm$ associated to eigenvalues $\lambda_1^\pm$. By tracing out the reference frame, these projectors induce a generalized measurement $\Lambda_1^\pm$ on the system, called a Positive Operator Valued Measurement (POVM) (see [22]). These can be compared with the idealized projective spin measurement induced by the kinematic operators $P_1^\pm = \frac{1}{2}(1 \pm \sigma_1)$.

As demonstrated in [8], we can write

$$\Lambda_1^\pm = \mathcal{V}(P_1^\pm), \tag{20}$$

where the *deformation map* $\mathcal{V}$ is some linear transformation that depends on the state of $\mathcal{R}_1$ and plays a role analogue to (2). The induced POVM and the kinematic projectors differ in two ways. First, they might be "misaligned" with one another, so $\mathcal{V}$ will in general contain a rotation. Moreover, $\Lambda^\pm$ may be a "noisy" version of $P^\pm$, e.g. $\Lambda^+ = (1-\epsilon)P^+ + \epsilon P^-$. In other words, the POVM may not perfectly distinguish between the subspaces P^+ and P^-, but may mix them up with some (not necessarily symmetric) probability ϵ. Thus, we will say that the transformation $\mathcal{V}$ contains a rotation and a mixing term. Due to the presence of the mixing term, the map $\mathcal{V}$ is *irreversible*. Only in a semi-classical regime, i.e. when $j \to \infty$ and $\|\langle \vec{J}_j \rangle\| \to 1$ will the deformation map be reversible.

As in the gravitational case, the transformation of the relational observables become non-linear when a "kick-back" is turned on between the particle and the reference frame. There are several ways to model this effect. In [20, 8], the authors have consider a scenario where the reference frame is used to successively measure a stream of particles in identical state $\rho_S \otimes \rho_S \otimes \ldots$. The average effect of a single measurement on the QRF can be described by the map [8]

$$\mathcal{N}_1(\langle\vec{\sigma}\rangle) : \vec{J}_j \to \sum_{a=\pm} \mathrm{Tr}_S\left(\Pi_1^a \left[\vec{J}_j \otimes \rho_S\right] \Pi_1^a\right), \tag{21}$$

where $\langle\vec{\sigma}\rangle = \mathrm{Tr}(\rho_S\vec{\sigma})$. After n such measurement, the state of the QRF will be transformed according to the map $\mathcal{N}_n(\langle\vec{\sigma}\rangle) = [\mathcal{N}_1(\langle\vec{\sigma}\rangle)]^n$. An other way to model the back-action is to assume that there is a weak Heisenberg coupling $H_I = \mu\sum_k \vec{\sigma}_k\cdot\vec{J}_j$ between each particle and the QRF. In turn, each particle scatter off the QRF for a finite amount of time after which the coupling is turned off. This scenario leads to essentially the same transformation $\mathcal{N}_n(\rho_S)$ of the QRF.

Since the stream of particle alter the QRF, they will also alter the relational observable themselves, i.e. the map $\mathcal{N}_n(\langle\vec{\sigma}\rangle)$ on $\vec{J}_j$ induces a map $\mathcal{U}_n(\langle\vec{\sigma}\rangle)$ on s_j:

$$\mathcal{U}_n(\langle\vec{\sigma}\rangle)(s_j) = \vec{\sigma}\cdot\mathcal{N}_n(\langle\vec{\sigma}\rangle)(\vec{J}_j) \tag{22}$$

which is the equivalent of (10). Due to the irreversibility of $\mathcal{V}$, we cannot carry on the analogy and express $\mathcal{U}_n$ as a non-linear function of the relational operators, e.g. $\mathcal{U}_n(\mathcal{V}^{-1}(\langle\vec{s}\rangle))(s_j)$. (Even if $\vec{\sigma}$ cannot be expressed as a function of $\vec{s}$, it is conceivable that $\mathcal{U}_n$ be expressible as a function of $\vec{s}$. We are currently investigating this possibility.) Thus, extra assumptions about the nature of the deformation map — namely reversibility — is required to complete the picture. This assumption is well justified in the semi-classical regime [8]. Just like the deformation map $\mathcal{V}$, the map $\mathcal{U}_n$ generally contains a rotation and a mixing component, so is also irreversible.

The transformation $\mathcal{U}_n$ is a non unitary map that could be seen as equivalent to U_{M_P} introduced in (10), with the key distinction that $\mathcal{U}_n$ is not invertible in general. This non invertibility is consistent with the initial motivation behind (1): when integrating out some degrees of freedom information is lost. Nonetheless, under reasonable circumstances (stronger than those leading to the reversibility of $\mathcal{V}$, see [8]), the mixing term of $\mathcal{U}_n$ can be negligible and the the map is effectively reversible.

Even if $\mathcal{U}_n$ is not invertible, we can still define the notion of deformed symmetries in the following sense. Consider another reference frame, $\mathcal{J}'$ yielding relational observables $\vec{s}'$ that are related to those obtained from reference frame $\mathcal{J}$ by a rotation $s'^j = R^j_i s^i$. When we turn on the interaction with the source of particles, the relation between these relational reference frames is non-linear $\mathcal{U}_n(\langle\vec{\sigma}\rangle)(s'^j) = \mathcal{U}_n(\langle\vec{\sigma}\rangle)(R^j_i s^i)$: the back-action induces a non-linear realization of the rotation group.

Conclusion

(Quantum) General Relativity is a constrained system and so observables are relational, i.e. constructed from (quantum) reference frames. With the help of a simple toy model, we illustrated that a deformation of the symmetry can naturally arise due to the back-action of the system on the reference frame. This back-action is unavoidable if the reference frame is to be used for its intended purpose of measuring particles, and can be explained by the creation of entanglement between the system and reference frame, c.f. (21). We also saw how the symplectic form is modified for the relational observables. As it should be the case when the full theory is accessible [2], our model did not show any ambiguity in the choice of symplectic form and physical observables.

The toy model is a good benchmark of the “modified measurement” approach introduced in [14] to interpret DSR. It suggests however that *caution should be taken when manipulating the deformation map as it is in general not invertible.* We find this aspect quite natural since our starting point (1) was a trace over the gravitational degrees of freedom, an operation which typically is accompanied by information loss.

Pushing the analogy with Quantum Gravity, this model supports the point of view that symmetry deformations of the type used in DSR are a natural candidate for the semi-classical limit, and gives no indication that a symmetry breaking should occur. From this perspective, the toy model provides a new and perhaps complementary mechanism to [14] (namely entanglement) to account for the system-dependent reference frame.

The toy model provides a natural and operational scheme to construct multiparticle states and to study the different meanings of the "momentum" addition. Following an operational approach, we obtained non-trivial multi-particle states and saw in particular that measuring the components of a system and adding the results yields different outcomes than measuring them as a whole. No satisfying equivalent construction is known for the "modified measurement" approach to DSR [14].

The toy model has of course its own limitations. Indeed it lacks of a universal constant playing the role of the Plank mass. In this sense it does not provide any insight into the problem of the bounded addition of the type (6). (In fact, this caveat is true for "modified measurement" approach as well.)

The difficulty of experimentally probing low energy QG is in great part responsible for the ambiguities associated to DSR, and other phenomenological approaches to effective QG. It may be interesting to pursue ideas along the lines presented here since they could eventually lead to experimental tests of the physics of deformed symmetries.

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